

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Even and Odd Functions

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\cot(-\theta) = -\cot \theta$$

Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Periodic Formulas

If n is an integer

$$\sin(\theta + 2\pi n) = \sin \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta$$

$$\tan(\theta + \pi n) = \tan \theta$$

$$\csc(\theta + 2\pi n) = \csc \theta$$

$$\sec(\theta + 2\pi n) = \sec \theta$$

$$\cot(\theta + \pi n) = \cot \theta$$

Cofunction Identities

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta)$$

$$\cos \theta = \sin(90^\circ - \theta)$$

$$\cot \theta = \tan(90^\circ - \theta)$$

$$\csc \theta = \sec(90^\circ - \theta)$$

Inverse Properties

$$\sin(\sin^{-1} x) = x \text{ for every } x \text{ in the interval } [-1, 1]$$

$$\sin^{-1}(\sin \theta) = \theta \text{ for every } \theta \text{ in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos(\cos^{-1} x) = x \text{ for every } x \text{ in the interval } [-1, 1]$$

$$\cos^{-1}(\cos \theta) = \theta \text{ for every } \theta \text{ in the interval } [0, \pi]$$

$$\tan(\tan^{-1} x) = x \text{ for every real number } x$$

$$\tan^{-1}(\tan \theta) = \theta \text{ for every } \theta \text{ in the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Double-Angle Formulas

$$\sin(2\theta) = 2\sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2\cos^2 \theta - 1$$

$$= 1 - 2\sin^2 \theta$$

$$\tan(2\theta) = \frac{2\tan \theta}{1 - \tan^2 \theta}$$

Power-Reducing Formulas

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Half-Angle Formulas

Sign determined by quadrant that half-angle lies

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$= \frac{1 - \cos \alpha}{\sin \alpha}$$

$$= \frac{\sin \alpha}{1 + \cos \alpha}$$

Product-to-Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum-to-Product Formulas

$$\sin \alpha + \sin \beta = 2\sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2\sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2\cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2\sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of an Oblique Triangle

$$\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's Formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c)$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(A-B)}{\sin \frac{1}{2}C}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(B-C)}{\tan \frac{1}{2}(B+C)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(A-C)}{\tan \frac{1}{2}(A+C)}$$

Length of a Circular Arc

$$s = r\theta$$